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TRANSFORMATION OF THE BENNET DISTRIBUTION IN A RAREFIED GAS

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The usual Bennet distribution of particles in a beam is characterized by a density which decreases comparatively slowly with increasing radius; this is a negative factor in the solution of problems associated with the formation of a thin, powerful, quasistationary, relativistic electron beam (REB). Under sufficiently high pressure of the residual gas the effect of the current of secondary electrons of the plasma, produced by the ionization of the gas by the beam, on the state of the main beam must be taken into account. As a result of this effect, an equilibrium state of the REB with a steeper, than in the case of the Bennet distribution, drop in the particle density toward the periphery of the beam can form.

For sufficiently high currents in the REB ($I \geq 1$ kA) the average Larmor radius of the electrons of the secondary plasma is shorter than the mean-free-path length λ_0 (magnetized diffusion) and the flow of secondary electrons acquires, owing to the magnetization, a longitudinal component [1]. The secondary flow can change the parameters of the REB, if the secondary-electron current is comparable to the beam current. In what follows we shall assume that the radius of the tube R_0 is much longer than the effective radius of the beam, the charge of the beam is completely compensated, the particle losses in the beam are negligibly small, and the frequency of collisions between plasma electrons and the gas is much higher than the electron-ion collision frequency. These assumptions are reasonable for $n_e \gg n_b$ and $n_g \gg n_e\sigma_c/\sigma_0$, where n_b and n_e are the density of electrons in the beam and in the plasma, n_g is the gas density ($n_g \leq 10^{15}$ cm^{-3}), σ_c is the Coulomb scattering cross section, and σ_0 is the cross section for scattering of electrons by gas atoms. We shall describe the secondary plasma by the hydrodynamic equations, and the primary beam by a system of self-consistent Vlasov equations.

In the kinetic description of a REB, the Bennet distribution is used most often. It can be established, for example, as a result of collisions of electrons in the beam with one another [2] or with particles in the medium [3].

For the distribution function of the electrons in the beam we shall use

$$f = \varkappa \exp \{-H/T + P_z/p_0\}, \quad (1)$$

where $H = c\sqrt{\bar{p}^2 + m^2c^2}$ is the Hamiltonian; $P_z = p_z + eA_z/c$ is the z component of the generalized momentum; $A_{(z)}(r)$ is the z component of the vector potential; $\bar{p}$ is the particle momentum; e and m are the charge and mass; and c is the velocity of light. The equilibrium nature of the state of the beam described is ensured automatically, if the component of the vector potential $A_z(r)$ satisfies Maxwell's equation

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$$\nabla\times\nabla\times\bar{A}_z = \frac{4\pi}{c}\sum_{(i)} j_z^{(i)}.$$

Calculation of the current density of the beam according to (1) yields the relation

$$j_z^{(1)} = \frac{I}{8\pi r_0^2}\, e^{\chi_z(r)}, \tag{2}$$

where I is the total beam current ($I = 2p_0c^2/e$); $r_0^2 = c\delta^2/(8\pi^2e^2\varkappa p_0^2)$ has the dimensions of length squares; $\chi_z(r) = eA_z/cp_0$; p_0 is a measure of the average transverse energy of the beam. If the condition $cp_0/T - 1 << \min(1, p_0/mc)$ holds, then for δ it is easy to obtain the simple expression $\delta \cong cp_0/T - 1$. The condition of convergence of the expression for $j^{(1)}_z$ requires that the inequality $cp_0 > T$ be strictly satisfied. When $\delta << 1$, the number of backward particles in the main beam is exponentially small.

To describe the state of the beam it is necessary to determine the longitudinal current of electrons in the secondary plasma. The plasma forms as a result of the ionization of the neutral gas atoms by the beam, and because of the uniformity of the system in the longitudinal direction the equation of continuity can be written in the form

$$\frac{1}{r}\frac{d}{dr}\left(rj_r^{(2)}\right) = n_g\sigma_i j_z^{(1)}, \tag{3}$$

where σ_i is the ionization cross section, $j^{(2)}_r$ is the radial component of the current density of secondary electrons; and $n^{(2)}$ is the particle density. Here we neglect the burnup of the neutral gas (n_g = const). Furthermore, we shall assume that the motion of the secondary electrons is diffusive (the state of magnetized diffusion [1]). This motion, neglecting the Coulomb friction, is described by the equation

$$-\frac{1}{n^{(2)}}\nabla\left(n^{(2)}T_e\right) + e\bar{E} + \frac{e}{c}\left[\bar{v}^{(2)}, \bar{B}\right] + \bar{F}_{\text{fr}}^{e,0} = 0,$$

where $n^{(2)}T_e$ is the pressure of the secondary electrons; $\bar{F}^{e,0}{}_{fr}$ is the force of friction between the electrons and the neutral gas; vT_e is the thermal velocity of the secondary electrons; λ_0 is the mean-free-path length; and $\bar{v}^{(2)}$ is the mean hydrodynamic velocity. The total magnetic field has only an azimuthal component, so that for the longitudinal component of the secondary-electron current we obtain

$$j_z^{(2)} = \alpha B_\theta j_r^{(2)} = \omega(r)\,\tau j_r^{(2)}, \tag{4}$$

where $\alpha = e\lambda_0/(mcvT_e)$; $\omega(r)\tau$ is the magnetization parameter.

In order to make use of the averaged hydrodynamic description of secondary electrons the Larmor radius $r_L \sim vT_e/\omega_B$ must be much shorter than the characteristic radius of the beam r_0. It is not difficult to obtain the estimate $r_L/r_0 \sim vT_e/2ic$, $i = eI/mc^3$. Thus, the dimensionless beam current i must be quite high.

The equation for the dimensionless longitudinal component of the vector potential has the form

$$\frac{1}{r}\frac{d}{dr}r\frac{d\chi_z}{dr} = -\frac{4\pi e}{p_0c^2}\left(j_z^{(1)} + j_z^{(2)}\right), \tag{5}$$

and, in addition, according to (4),

$$\frac{e}{cp_0}j_z^{(2)} = -\alpha\frac{d\chi_z}{dr}j_r^{(2)}.$$

From these relations it follows that

$$j_r^{(2)} = \frac{\frac{1}{r}\frac{d}{dr}r\frac{d\chi_z}{dr} + \frac{4\pi}{c}j_z^{(1)}}{4\pi\alpha\frac{p_0}{e}\frac{d\chi_z}{dr}},$$

whence, with the use of (2) and (3), we find the equation for determining $\chi_z(r)$:

$$\frac{d}{dr}\frac{\frac{d}{dr}r\frac{d\chi_z}{dr}+\frac{r}{r_0^2}e^{\chi_z}}{\frac{d\chi_z}{dr}}=\alpha^*\frac{r}{r_0^2}e^{\chi_z}, \tag{6}$$

where $\alpha^*=n_g\sigma_i\lambda_0\frac{p_0}{v_{T_e}}=\frac{ic}{2v_{T_e}}\frac{\sigma_i}{\sigma_0}$ (here the relation $\lambda_0 = 1/n_g\sigma_0$).

For $\alpha^* < 1/2$, Eq. (6) has a solution of the form

$$\chi_z=-2\ln\left[1+\frac{r^2}{8r_0^2}(1-2\alpha^*)\right], \tag{7}$$

which corresponds to the differential form of (6),

$$\frac{d}{dr}r\frac{d\chi_z}{dr}+\frac{r}{r_0^2}e^{\chi_z}=-\frac{\alpha^*}{r_0^2}\frac{d\chi_z}{dr}\int_r^\infty e^{\chi_z(r')}r'dr'.$$

The right side of this equation is the secondary-electron current directed opposite to the current in the main beam, i.e., having a "demagnetizing" character. It is evident from the relation (4) that if the sign of the current $j^{(2)}_z$ is negative, then the sign of the current $j^{(2)}_r$ will also be negative, i.e., the secondary electrons will move toward the axis, where in this case a sink must be provided for the particles (for example, in the form of a very thin conductor).

We shall study a situation which is more interesting from the physical viewpoint, when the secondary particles are drained at the walls of the tube, situated at quite a large distance away from the axis of the beam. In this case, the reverse current also flows along the walls of the tube. Introducing the dimensionless independent variable $x = r^2/r_0^2$, we rewrite (6) in the integrodifferential form

$$4\frac{d}{dx}x\frac{d\chi_z}{dx}+e^{\chi_z}=\alpha^*\frac{d\chi_z}{dx}\int_0^x e^{\chi_z(x')}dx'. \tag{8}$$

The choice of the limits of integration on the right side corresponds to dispersing of electrons away from the axis of the beam; in addition, the relation (7) does not satisfy Eq. (8).

An equation equivalent to (8) can be simply derived using the hydrodynamic description. We take the system (3)-(5) (using $B_\theta=-\frac{cp_0}{e}\frac{d\chi_z}{dr}$) and supplement it by the equation of balance of the forces acting on the particles in the beam in the radial direction:

$$\frac{1}{n}\frac{dQ}{dr}=-e\beta_0 B_\theta. \tag{9}$$

Here Q(r) is the pressure of the particles in the beam with a Bennet distribution $Q = nT$, $\beta_0 = v_z/c$, and the mean hydrodynamic velocity $v_z(r) = \text{const}$.

From (3)-(5) it is easy to obtain

$$\frac{c}{4\pi}\frac{d}{dr}rB_\theta-rj_z^{(1)}=(\alpha n_g\sigma_i)B_\theta\int_0^r r'dr'j_z^{(1)}(r').$$

From (9) it follows that

$$B_\theta=-\frac{T}{c\beta_0}\frac{1}{j_z^{(1)}}\frac{d}{dr}j_z^{(1)}.$$

Eliminating B_θ from these relations, we obtain an equation for n which is equivalent to (3). We shall solve (8) with the boundary condition

$$\chi_z(0) = d\chi_z/dr|_{r=0} = 0. \tag{10}$$

The equation contains the dimensionless parameter α^*, whose values can vary over a wide range. In particular, for values of the parameters close to the real values (see [1]), $\sigma_i = 10^{-18}$ cm^{-2}, $\sigma_0 = 10^{-15}$ cm^{-2}, $T_e \sim 0.1\text{-}1$ eV, $\alpha^* \leqq i$. The condition that the Larmor radius of the secondary electrons be small for the given values of the parameters acquires the form I >> 1.7-17 A, i.e., it is obviously met in the range of values of I under study. For $\alpha^* = 0$, the solution of (8), satisfying the conditions (10), is

$$\chi_z = -2\ln\left(1 + \frac{r^2}{8r_0^2}\right),$$

which corresponds to the usual Bennet distribution.

It is not difficult to verify that for $\alpha^* = 1$ the solution of (8) with the condition (10) is

$$\chi_z = -x/4.$$

If in the case of the usual Bennet distribution the dependence of the longitudinal current density on the radius is

$$j_z^{(1)} = \frac{1}{8\pi r_0^2}\,\frac{1}{(1 + r^2/8r_0^2)^2},$$

then in the case described by the solution found the distribution for the same values of I and $r_0{}^2$ is Gaussian:

$$j_z^{(1)} = \frac{I}{4\pi r_0^2}\exp\left(-r^2/4r_0^2\right),$$

i.e., the current density on the axis is two times higher than the Bennet density and decreases very rapidly exponentially in the radial direction. This fact apparently can play a large role for the transport of a high-current beam in the drift space, since the Bennet distribution has long "tails" (the rms radius diverges), while for the Gaussian distribution which we have found all average values converge. It is, however, impossible to estimate the change in the radius of the beam under real conditions on the basis of the given stationary problem if the beam current I may be assumed to be conserved, which cannot be said about the parameter $r_0{}^2$, because the transformation of the beam from a Bennet beam to a Gaussian one is a dynamic process.

The value of the parameter $\alpha^* = 1$ is attainable with very high currents of the primary beam: $I \leqq 17$ kA. We shall study the region of values of the parameter $\alpha^* \ll 1$, which corresponds to a small perturbation of the Bennet distribution of the beam by the magnetic field generated by the secondary electron current in the plasma.

Setting

$$\chi_z(r) = \chi_{z0}(r) + \alpha^*\chi_{z1}(r), \tag{11}$$

substituting (11) into (8), and retaining terms no higher than first-order infinitesimals in α^*, we obtain

$$4\frac{d}{dx}x\frac{d\chi_{z1}}{dx} + \frac{\chi_{z1}}{(1 + x/8)^2} = -\frac{x}{4}\,\frac{1}{(1 + x/8)^2}. \tag{12}$$

Substituting $\xi = (x - 8)/(x + 8)$ (12) is reduced to an inhomogeneous Legendre equation, and its solution, satisfying the conditions (10), has the form

$$\chi_{z1} = -\frac{3x}{2(1 + x/8)} + 8\ln(1 + x/8) - 4\frac{x - 8}{x + 8}\int_0^{x/8}\frac{\ln(1 + t)}{t}\,dt, \tag{13}$$

whence it follows that near the axis the current density in the beam decreases with the increasing radius more slowly than in the usual Bennet distribution:

$$j_z^{(2)}(r) \sim 1 - \frac{r^2}{4r_0^2} + \frac{r^4}{64r_0^4}\alpha^*.$$

Solution (13) is valid for bounded values of r, such that the inequality

$$\alpha^* \chi_{z1}(r) < \chi_{z0}(r)$$

is satisfied. Under the conditions of a real experiment during propagation of a beam in a waveguide of finite radius this restriction is not very significant. Comparison of the solutions with $\alpha^* = 1$ and $\alpha^* \ll 1$ shows that the effect of the longitudinal electron current in the secondary plasma gives rise to an equilibrium state of the REB, characterized by a steeper, than with the Bennet distribution, drop in the current density toward the periphery of the beam. As the parameter α^* increases, the slope of the dropoff increases, and at $\alpha^* = 1$ the beam transforms into a Gaussian beam. At the same time it is not difficult to obtain the distribution of the secondary particle current. For the radial component of the density

$$j_r^{(2)} = \frac{n_g \sigma_i I}{2\pi r}\left(1 - e^{-r^2/4r_0^2}\right).$$

The radial current vanishes on the axis of the beam, has a maximum at $r \sim r_0$, and drops off as $\sim 1/r$ in the limit $r \to \infty$. For the longitudinal component

$$j_z^{(2)} = \frac{p_0 \sigma_i I}{4mv_{T_e}\pi r_0^2 \sigma_0}\left(1 - e^{-r^2/4r_0^2}\right).$$

Setting $p_0 \sim mc$, we obtain the total longitudinal current of secondary electrons, flowing in the beam ($r \leqq r_0$):

$$I^{(2)} \simeq \frac{c}{v_{T_e}} \frac{\sigma_i}{\sigma_0} I.$$

For the values of the parameters presented above $I^{(2)} \sim I$.

Thus, the main result of this work is that for REB currents of $I \geqq 1$ kA the longitudinal electron current of the secondary plasma significantly alters the configuration of the electron base.

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